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06MAT41

Fourth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Applying Taylor's series method, find 'y' at x = 0.1 by considering the series up to fourth degree term. Given $\frac{dy}{dx} = x + y^2$, y(0) = 1. **(06 Marks)**

b. Find y at x = 0.2 using Runge Kutta fourth order method. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 and step size 0.2. **(07 Marks)**

c. Use Milne's predictor corrector formulae to find y(0.4). Given $\frac{dy}{dx} = y(x+y)$, with

x	0	0.1	0.2	0.3
y	1	1.1169	1.2733	1.5049

(Use corrector formula twice.) **(07 Marks)**

2 a. If f(z) in a regular function of 'z' show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. **(06 Marks)**

b. Find the analytic function f(z) = u + iv where $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$. **(07 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, -1 into the point w = 2, i, -2. Also find the invariant points of the transformation. **(07 Marks)**

3 a. State and prove Cauchy's theorem under complex values functions integration. **(06 Marks)**

b. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent's series valid for (i) |z| < 1, (ii) 1 < |z| < 2. **(07 Marks)**

c. Evaluate $\int_c \frac{z\cos z}{(z-\pi/2)^3} dz$ where 'c' is |z – 1| = 1 using Cauchy's residue theorem. **(07 Marks)**

4 a. Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ using the method of solution in series. **(06 Marks)**

b. With usual notation prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(07 Marks)**

c. Express $x^4 - 3x^2 + x$ in terms of Legendre's polynomials. **(07 Marks)**

PART – B

5 a. Fit a curve in the form y = a + bx + cx² by the method of least squares for the data given below:

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.3	1.6	2	2.7	3.4	4.1

(06 Marks)

5 b. Obtain the two lines of regressions for the data given below:

x	1	2	3	4	5
y	2	5	3	8	7

and hence find the coefficient of correlation between x and y. **(07 Marks)**

c. The contents of the three urns are as given below:
Urn 1: 1 white ball, 2 red balls, 3 green balls.
Urn 2: 2 white, 1 red and 1 green ball.
Urn 3: 4 white, 5 red and 3 green balls.
Two balls are drawn from a randomly chosen urn and are found to the one white and one green. Find the probability that the balls so drawn came from the third urn. **(07 Marks)**

6 a. Find the mean and variance of Poisson distribution. **(06 Marks)**

b. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for (i) 10 minutes or more, (ii) less than 10 minutes, (iii) between 10 and 12 minutes? **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation of 5. Find the expected number of students, whose marks will be:
i) less than 65 ii) more than 75 iii) between 65 and 75
[Given A(z = 1) = 0.3413]. **(07 Marks)**

7 a. A coin is tossed 1000 times and head turned up 540 times. Test the hypothesis that the coin is an unbiased one. **(06 Marks)**

b. A sample of 200 tyres is taken from a lot. The mean life of tyres is found to be 40000 kms with standard deviation of 3200 kms. Is it reasonable to assume the mean life of tyres in the lot as 41000 kms? Also establish 95% confidence limits with in which the mean life of tyres in the lot is expected to lie. **(07 Marks)**

c. Ten individuals are chosen at random from a population and the heights are found to the in inches 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. In the light of this data discuss the suggestion that the mean height in the population is 66 inches. If the population mean is unknown, obtain 90% confidence limits for this mean. **(07 Marks)**

8 a. The joint probability distribution for two random variables x and y is given below:

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find the (i) marginal probability distributions of x and y, (ii) covariance of x and y. **(06 Marks)**

b. Find the unique fixed probability vector of the regular stochastic matrix

$$P = \begin{bmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(07 Marks)

c. A software engineer goes to his work place every day by motor bike or by car, he never goes by bike on two consecutive days, but if he goes by car on a day then he is equally likely to go by car or by bike on the next day. Find the transition matrix for the chain of the mode of transport. If car is used on the first day of a week find the probability that (i) bike is used, (ii) car is used on the 5^{th} day. **(07 Marks)**

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